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On the definition of the power coefficient of tidal current turbines and efficiency of tidal current turbine farms

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ABSTRACT

During the last decade, the development of tidal current industries has experienced a rapid growth. Many devices are being prototyped. For various purposes, investors, industries, government and academics are looking to identify the best device in terms of of cost of energy and performance. However, it is difficult to compare the cost of energy of new devices directly because of uncertainties in the operational and capital costs. It may however be possible to compare the power output of different devices by standardizing the definition of power coefficients. In this paper, we derive a formula to quantify the power coefficient of different devices. Specifically, this formula covers ducted devices, and it suggests that the duct shape should be considered. We also propose a procedure to quantify the efficiency of a tidal current turbine farm by using the power output of the farm where no hydrodynamic interaction exists between turbines, which normalizes a given farm's power output. We also show that the maximum efficiency of a farm can be obtained when the hydrodynamic interaction exists.

1. Introduction

Tidal energy has been utilized since Roman times, or before [1], when ancient people put their tidal mills in the water to harness energy by utilizing the elevation change of the tide. This resource is called tidal range and the technology used to harness the energy is called the tidal barrage. However, during the last few decades, this technology has not been used much to extract energy because of its low efficiency and high environmental impact. In late 1990s', the tidal energy gained the attentions again with a significant change of the energy conversion technology. The energy converter changed to underwater turbine which is analog to wind turbine, a successful technology to generate energy from air flow (Fig. 1), and this resource is called tidal current. A few companies have deployed their design in full-scale in the sea. Because these designs are approaching the commercial stage, several governments have begun to focus on identifying the most promising device for market acceleration. Technological investigations are being pursued to help facilitate commercialization and support industry growth [2,3]. Similarly, some private investors are also trying to identify the best device to evaluate potential investment opportunity [4]. Additionally, researchers are optimizing the devices to reach the cost-effectiveness from an engineering point of view [5,6].

To determine whether a turbine is worth investigating, the project developer usually uses the cost of energy to check the cost-effectiveness of a turbine or turbine farm. The cost of energy is defined as the ratio of the total cost to the total energy output over the lifetime of a turbine or farm. Mathematically, it can be estimated by using Eq. (1).

$$c_{\text{energy}} = \frac{\sum_j \text{levco}_j}{\sum_j \text{Energy}_j} \tag{1}$$

where levco_j and Energy_j denote the levelized cost (present value of the total cost of building and operating a power plant over its economic lifetime) and the energy output in the year j, respectively. The levelized cost is directly determined by the turbine materials and operational strategies [7]. Because the tidal current industry is still developing new turbine materials as well as operational strategies, it is difficult to evaluate the cost of energy. Thus, it can be more productive to study the total energy output, which is expressed as follows,

$$\text{Energy} = \mathbb{E}(P(t), f, T) \tag{2}$$

where $\mathbb{E}$ denotes the function of calculating the total energy output, P denotes the power output, t denotes instant time, f denotes the electric conversion efficiency, and T denotes the lifetime of the device or farm.

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Fig. 1. Examples of turbines: (a) wind turbine (at NREL) and (b) tidal current turbine (courtesy by Verdant power).

As the energy output is highly site dependent, researchers always turn to focus on the power output. In order to standardize this discussion, researchers quantify the power output of the turbine by analyzing its dimensionless format, power coefficient. However, there is currently no universally accepted definition of power coefficient for tidal current turbines because dozens of new prototypes with nontraditional designs have emerged during the past decade. The traditional definition does not consider the auxiliary structures such as duct and flapping foil. Consequently, it is difficult to use the traditional definition to evaluate the cost-effectiveness of a nontraditional turbine system. In order to provide a precise method to quantify the power output of different designs, we propose a new way to calculate the power coefficient that applies to both traditional and nontraditional turbines, and we summarize the effort in this paper. Specifically, after reviewing the traditional turbine design, we discuss the difference between the design of the nontraditional turbines and the design of traditional turbines. Then, we propose a new reference power to calculate power coefficient that handles both the nontraditional turbines and traditional turbines. Examples of nontraditional turbines are shown, as well as a procedure to quantify the farm efficiency for the purpose of resource assessment and farm planning. To obtain the farm efficiency, we suggest that one shall use the power output of the farm where no hydrodynamic interaction exist between turbines as the reference power to normalize a farm power output. Finally, we discuss the limitations of the new methods.

2. Definition of turbine power coefficient

The turbine power coefficient is defined as the actual power output divided by a certain reference power output. For the simplicity of future discussion, we define a parameter, P_{ref}, as the reference power which is used to nondimensionalize the actual power output; consequently, the power coefficient of a generic turbine can be given as Eq. (3). This reference power output is determined by the geometry of the turbine and the characteristics of the inflow.

$$C_P = \frac{P}{P_{\text{ref}}} \tag{3}$$

2.1. Traditional turbine

For traditional turbines, i.e., open water vertical-axis turbines and horizontal-axis turbines (Fig. 2), the power output is nondimensionalized by the maximum power output that can be generated from the kinetic energy flux of a free stream flow through the turbine projected frontal area, the velocity of which is uniform in space and constant in time. Theoretically, for such an undisturbed free stream flow through $1/2\rho AU_\infty^2$ (where ρ, A and U_∞ denote the water density, the frontal area of the turbine and the far-field incoming flow velocity, respectively), we can have the maximum power output as $1/2\rho AU_\infty^3$. Therefore, to evaluate the power coefficient of a traditional turbine, the reference power for the traditional turbine can be given as,

$$P_{\text{ref,trad}} = \frac{1}{2}\rho AU_\infty^3 \tag{4}$$

Thus, we have what we often see in textbooks and articles about the turbine power coefficient,

$$C_P = \frac{P}{\frac{1}{2}\rho AU_\infty^3} \tag{5}$$

2.2. Nontraditional

Recently, quite a few nontraditional designs have been proposed and built, among which ducted turbines most popular.[1] Some have been developed by companies such as Lunar Energy, Clean Current and Open Hydro and some are developed by universities such as the University of Buenos Aires [8], and the University of British Columbia [9]. The main purpose of using the duct is to augment the flow passing through the turbine so as to increase the power output. The more optimal the duct profile is, the higher the turbine power output is. Therefore, it is not always suitable to use the traditional power coefficient definition, Eq. (5) to dimensionalize the power output because of the definition of the frontal area, A, and the incoming flow velocity U_∞. Some suggest that the frontal area shall be kept as the original front area [11], i.e.,

$$A = \pi r^2 \tag{6}$$

where r denotes the turbine radius in the duct. Some suggest that the frontal area shall be the frontal area of the duct [12], i.e.,

[1] Here, duct refers to the shroud structure around the turbine (see Fig. 3). Those large structure around turbine such as dam or barrage type are not considered here. They are beyond the scope of this paper and won't be discussed.

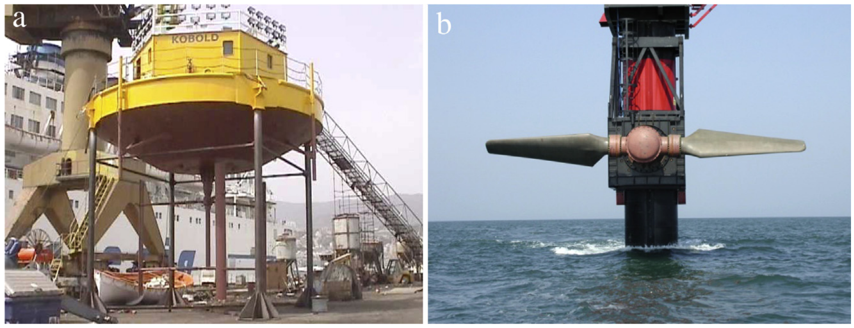

Fig. 2. Classification of turbines: a) vertical axis turbine (courtesy of Prof Coiro from University Naples) and b) horizontal axis turbine (courtesy of Peter Fraenkel from Marine Current Turbine).

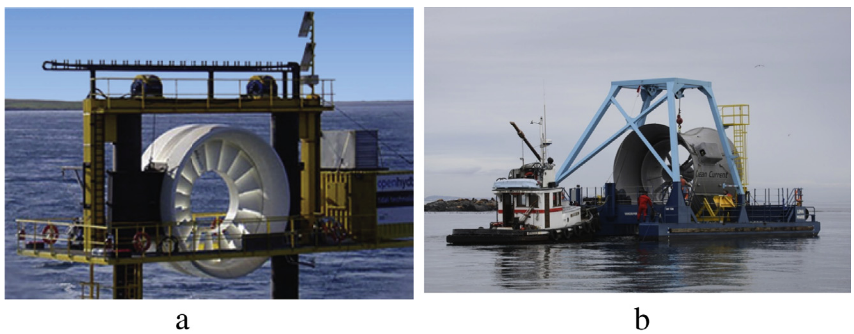

Fig. 3. Nontraditional turbines: a) Open Hydro (courtesy of Open Hydro), and b) Clean Current (Courtesy of Clean Current).

$$A = \mathbb{A}(-L_1) \tag{7}$$

where $\mathbb{A}$ denotes the function of calculating the cross-section area of duct, and L_1 denotes the location of the beginning of the cross-section of the duct (see Fig. 4). For example, the cross-section area of a typical ducted horizontal tidal current turbine where the shape of the cross-section is circle can be obtained as follows,

$$\mathbb{A}(x) = \pi r^2(x) \tag{8}$$

However, the above definitions just quantify the efficiency in one-dimension at one cross-section, because the power coefficient formulation is the same as the one for traditional turbines. Such a definition might be useful after the design has been finalized for a ducted horizontal axis turbine, including both geometry of the system and the turbine location with respect to the duct. However, in a ducted turbine design process, the turbine is usually designed first, then the duct is designed, and finally the location for the turbine within the duct is determined [8,9]. As the cross-section of the duct varies along its length, and the length of the ducted turbine is much longer than the turbine, strong rational may not exist for locating the turbine in a specific cross-section of the duct. Therefore, it is better to quantify reference power for the entire duct, in a manner that account for potential variation of the turbine axial location in two-dimensions.[2] Furthermore, for vertical turbines or cross flow turbines, the blades work in different axial location of the duct. The inflows they encounter are different for every blade. Therefore, there needs to be a method that quantifies the efficiency and considers the duct profile change. In short, we believe that a new reference power is required to fully consider the change of the duct profile. We tentatively propose to integrate the flow flux in the duct in the incoming flow direction to estimate the reference power output.[3] Mathematically, by assuming the flow is uniform everywhere in any cross-section, we define the new reference power as following (refer to Fig. 4),

$$P_{\text{ref}} = \frac{1}{2(L_1+L_2)}\rho \int_{-L_1}^{L_2} \mathbb{A}(x)U^3(x)\mathrm{d}x \tag{9}$$

where L_1 and L_2 and $U(x)$ denote the location of the ends of the cross-section of the duct and the flow velocity at axial location x, respectively. Based on the mass conservation law, we can obtain $U(x)$ using Eq. (10).

$$U(x) = \frac{\mathbb{A}(-L_1)U_\infty}{\mathbb{A}(x)} \tag{10}$$

By substituting Eq. (10) into Eq. (9), we can obtain the new reference power as Eq. (11),

$$P_{\text{ref}} = \frac{\mathbb{A}^3(-L_1)U_\infty^3}{2(L_1+L_2)}\rho \int_{-L_1}^{L_2} \frac{1}{\mathbb{A}^2(x)}\mathrm{d}x \tag{11}$$

Therefore, by substituting Eq. (11) into Eq. (3), the new power coefficient can be written as

[2] One can do a three-dimension treatment if the duct is asymmetric. In this article, we refer to the symmetric duct with symmetric turbine, of which three-dimensional effect is not significant [10]. Therefore, here our treatment is two-dimension only.

[3] The proposal here is to show a possible method to quantify the power coefficient of ducted turbine; it may not be the best way and it still has deficiencies as discussed in this paper, but we hope it can help the designer to avoid some introductory level mistakes.

$$C_P = \frac{P}{\frac{\mathbb{A}^3(-L_1)U_\infty^3}{2(L_1+L_2)} \int\limits_{-L_1}^{L_2} \rho \frac{1}{\mathbb{A}^2(x)} \mathrm{d}x} \tag{12}$$

Here we give an example of using above formulation, Eq. (12), to study the power coefficient of a nontraditional turbine. We evaluate a ducted turbine with a symmetric parabolic duct with a horizontal turbine inside. Parabolic boundary has been widely used in the ocean engineering field for optimizing the performance of ships [13]. Mathematically, we define the shape of the duct as shown in Fig. 4 and with Eqs. (13) and (14).

$$l = ax^2 + r \tag{13}$$

$$L_1 = L_2 = L \tag{14}$$

where l and a denote distance between turbine center and the duct profile parameter, respectively. Therefore, by substituting Eqs. (13) and (14) into Eq. (12) and after some mathematical derivations, we can obtain the reference power as Eq. (15). In order to keep the continuity of the discussion here, we leave the detailed derivations in Appendix.

$$P_{\mathrm{ref}} = \rho\pi \frac{(aL^2+r)^6 U_\infty^3}{8L} \left(\frac{5 \times \arctan\left(L\sqrt{\frac{a}{r}}\right)}{8r^{3.5}\sqrt{a}} + \frac{5L}{8r^3(r+aL^2)} + \frac{5L}{12r^2(r+aL^2)^2} + \frac{L}{3r(r+aL^2)^3} \right) \tag{15}$$

Thus, the new power coefficient of this turbine can be obtained as Eq. (16),

$$C_P = \frac{P}{\rho\pi \frac{(aL^2+r)^6 U_\infty^3}{8L} \left(\frac{5\times\arctan\left(L\sqrt{\frac{a}{r}}\right)}{8r^{3.5}\sqrt{a}} + \frac{5L}{8r^3(r+aL^2)} + \frac{5L}{12r^2(r+aL^2)^2} + \frac{L}{3r(r+aL^2)^3} \right)} \tag{16}$$

We compare the power coefficients of ducted turbines calculated by all three different definitions[4]: 1) traditional definition considering the turbine frontal area as given by Eq. (6); 2) alternative definition considering the duct frontal area as given by Eq. (7); and 3) the new definition proposed in this paper by considering the whole duct shape as given by Eq. (16). We calculate them with various parabolic parameters. In this comparison, we assume that the turbine's power output is independent from the parabolic profile. Furthermore, the power coefficient obtained with Eq. (6), by its definition, is independent from the existence of the duct. It is a constant in this calculation, and we assume it 50%.

Therefore, all the power coefficients are no more than 50% (Fig. 5). It is noted that the power coefficients obtained with Eq. (16) and with Eq. (7) both decrease when the length to radius ratio increases. This indicates that it is not efficient to take a larger space by increasing the duct length. The results also show that the power

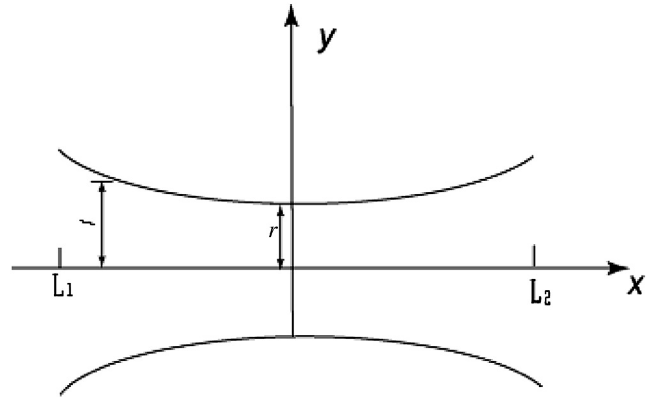

Fig. 4. Illustration of the parabolic duct.

coefficient obtained with Eq. (16) is smaller than that obtained with Eq. (7), and the power coefficient obtained with Eq. (7) is smaller than that obtained with Eq. (6). The difference between these two is larger when a is larger. This is because that Eq. (16) considers the curvature of the duct while Eq. (7) does not. Particularly, when the length to radius ratio is equal to 0.25 and a is equal to one, the new power coefficient is about 24% less than the traditional definition that does not consider the duct. That is, if we do not consider the duct shape, a designer can over report the power coefficient of a turbine system by optimizing the duct and taking a large amount of space.

When using this new definition to evaluate different duct shapes, one needs to be very careful to assume that the power outputs of both turbines are the same. Here, we give an example of using the new power coefficient to evaluate two different duct shapes with a same frontal area, and we assume that the power outputs of both turbines are the same. We consider one turbine with a parabolic duct where a is equal to 0.5 and the other turbine with a linear duct. The radius of the cross-section of the linear duct can be expressed as Eq. (17).

$$l = bx + r \tag{17}$$

With the same mechanism shown in Eqs. (11)–(16), by plugging Eq. (17) into Eq. (12), we can have the power coefficient of the linear duct as Eq. (18).

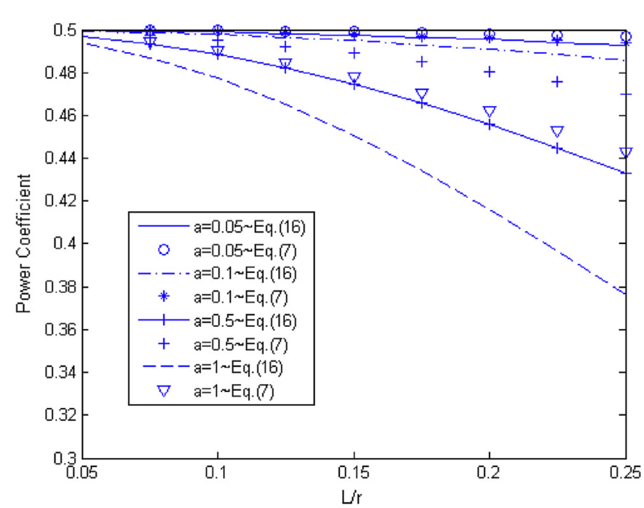

Fig. 5. A comparison between new power coefficient and existing power coefficient.

[4] For those who would like to compare the difference between each reference power, they can simply consider the reciprocal value of the power coefficient here.

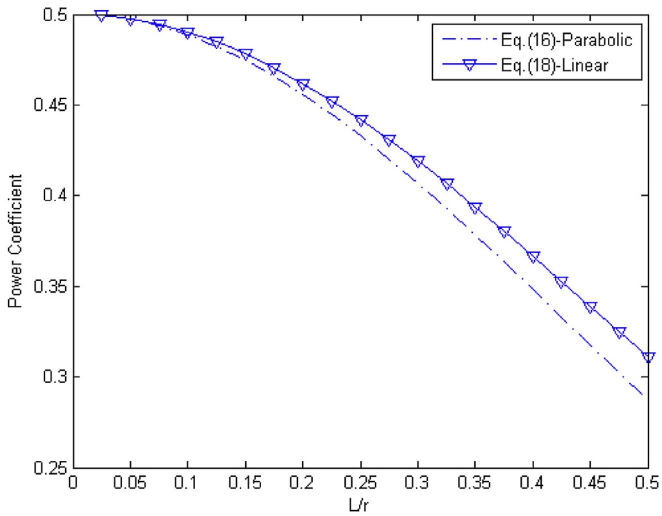

Fig. 6. A comparison between the power coefficients of a parabolic duct and a linear duct with the same duct frontal area.

$$C_P = \frac{P}{\rho\pi\frac{(0.5L^2+r)^6U_\infty^3}{3L^2}\left(\frac{1}{r^3}-\frac{1}{(0.5L^2+r)^3}\right)} \tag{18}$$

When the length to radius ration remains the same, the power coefficient of the linear duct is higher than that of the parabolic duct (Fig. 6). However, this does not suggest that the linear duct utilize the space more efficient than the parabolic duct. It is understood that, if the sizes are the same, the power output of the turbine with a parabolic duct is higher than that with a linear duct [9]. Therefore, as we assume their power outputs are the same, one can derive that the size of the turbine in the parabolic duct is smaller than that in the linear duct. More importantly, if we use the power coefficient obtained with Eq. (6) to evaluate these two turbine systems, we cannot make a judgment easily. In short, we find that new power coefficient obtained with Eq. (12) fully considers the space that the turbine system takes, although the illustrations here neglect the unsteady and uniform flow phenomenon in the duct. One can still make the conclusion that the new reference power definition is more appropriate for a ducted turbine.

3. Farm efficiency

The analysis in Section 2 discusses the relationship between a turbine's displacement and its power coefficient. In this section, we discuss the efficiency of a turbine farm that refers to the commercial scheme of a group of tidal current turbines in a site. The construction of a tidal current turbine farm is expected to require substantial investment. Consequently, it is necessary to evaluate the power output of the farm. Until now, there has been no universally-accepted way to quantify the farm efficiency.

Existing studies of the tidal current turbine farm's power output focus on energy potential for the resource assessment purpose. Consequently, these assumptions follow the same purpose without the details of the turbines. For example, Triton [14] assumes that the flow through a certain cross-section of the channel can be fully utilized to generate power (this approach assumes that multiple turbines can utilize the flow everywhere in this channel, i.e., there is no distance between any two-neighboring turbines), so this power is used to represent the maximum power output of the whole channel. Garret and Cummins [15] studied the maximum power output of a channel from an oceanography point of view by treating the turbine as a black box and the channel as a two-dimensional flow with lateral boundary. Whelan et al., [16] followed Garret and Cummins [15] and studied the maximum power output of a channel by treating the channel as a two-dimensional flow with vertical boundaries. These studies may help energy analysts to estimate the energy potential. For example, Polagye et al., [17] improved the Triton [14] method with a one-dimensional model to estimate the tidal energy potential in Puget Sound. Karsten et al., [18] used the method developed by Garret and Cummins [15] to estimate the tidal energy potential in the Bay of Fundy. However, these approaches cannot provide engineering criteria for farm planners.

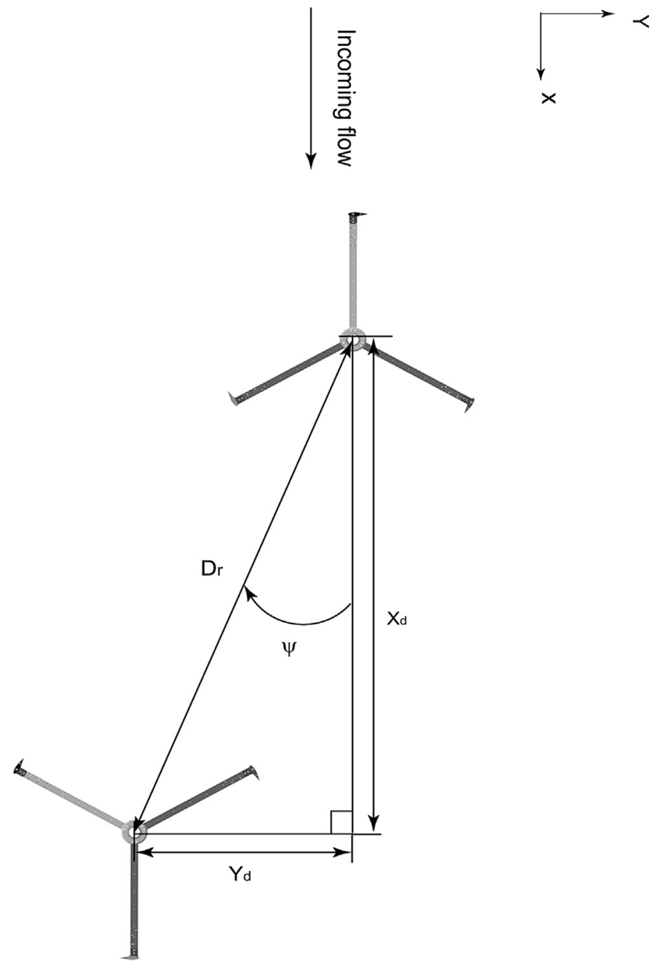

Fig. 7. An illustration of the incoming flow angle and the relative distance of a twin-turbine system (adapted from Li and Calisal [19]).

One important objective for the farm designer is to maximize the total power coefficient. If the number of turbines in the farm is only one, i.e., the turbine is stand-alone, the displacement of the turbine is not critical. However, due to the nature of the operationing conditions and the need for cost-effective power plants, the construction scheme for a tidal current turbine farm is expected to have turbines that are closely spaced. When these turbines are close to each other, there are hydrodynamic interactions affecting the performance of each turbine. Thus, we need to evaluate the power output of individual turbines in an N-turbine system with that of the corresponding stand-alone turbine. Here, similar to the single turbine power coefficient, we define a farm efficiency, η, as

$$\eta = \frac{P_{\text{farm}}}{P_{\text{Ref,farm}}} \tag{19}$$

$$P_{\mathrm{Ref,farm}} = N \times P_S \tag{20}$$

$$P_{\mathrm{farm}} = \sum_{i=1}^{N} P_i \tag{21}$$

where P_S denotes the power output from the corresponding stand-alone turbine, P_i denotes the power output of turbine i, and $P_{\mathrm{Ref,farm}}$ denotes the reference power output of the farm at given operational condition. All power output variables here represent the mean power output under optimal condition, i.e., the maximum mean power output. Particularly, $P_{\mathrm{Ref,farm}}$ does not only represent summation of the power output of N stand-alone turbine but also, more practically, it represents the power output of the farm where no hydrodynamic interaction exists between the turbines.[5] This is the same as using the maximum power from a free stream continuous flow without a turbine as the reference power output for the turbine power coefficient as discussed in Section 2. We call the farm where no hydrodynamic interaction exists, the reference farm. In the reference farm, any two neighboring turbines are spaced at a distance where no hydrodynamic interaction exists between the turbines. This distance is defined as effective distance, d_e and it can be expressed as Eq. (22). In reality, a farm planner may have a farm with turbine distance less than the effective distance due to economic consideration as aforementioned. The corresponding turbine distribution brings a different power output from the reference farm, so the hydrodynamic interaction between turbines is key.

$$d_e = \mathbb{F}(\psi, D_r, \mathrm{TSR}, \text{turbine design parameters}, U_\infty, \text{relative rotational direction}) \tag{22}$$

where ψ denotes the incoming flow angle and D_r denotes the relative distance, which is the distance between the shafts of the two turbines as shown in Fig. 7. Mathematically, they can be obtained as follows,

$$\psi = \tan^{-1} Y_d / X_d \tag{23}$$

$$D_r = \sqrt{\left(X_d^2 + Y_d^2\right)} \tag{24}$$

$$\begin{cases} X_d = X_{\mathrm{up}} - X_{\mathrm{down}} / R \\ Y_d = Y_{\mathrm{up}} - Y_{\mathrm{down}} / R \end{cases} \tag{25}$$

where, X_d and Y_d denote the relative distance between two turbines in the x and y directions, respectively, R denotes the radius of an individual turbine, and (X_{up},Y_{up}) and (X_{down},Y_{down}) indicate the positions of the upstream turbine and the downstream turbine, respectively. Additionally, relative rotational direction in Eq. (22) refers to the rotational direction of the turbines. Any two turbines can be operated in two different relative rotating directions, either co-rotating, which means that both turbines rotate in the same direction (either clockwise or counterclockwise), or counter-rotating, which means that two turbines rotate in the opposite direction with one being clockwise and the other being counterclockwise. Turbine design parameters refer regular parameters of a turbine such as the blade profiles, radius, height and pitch angle.

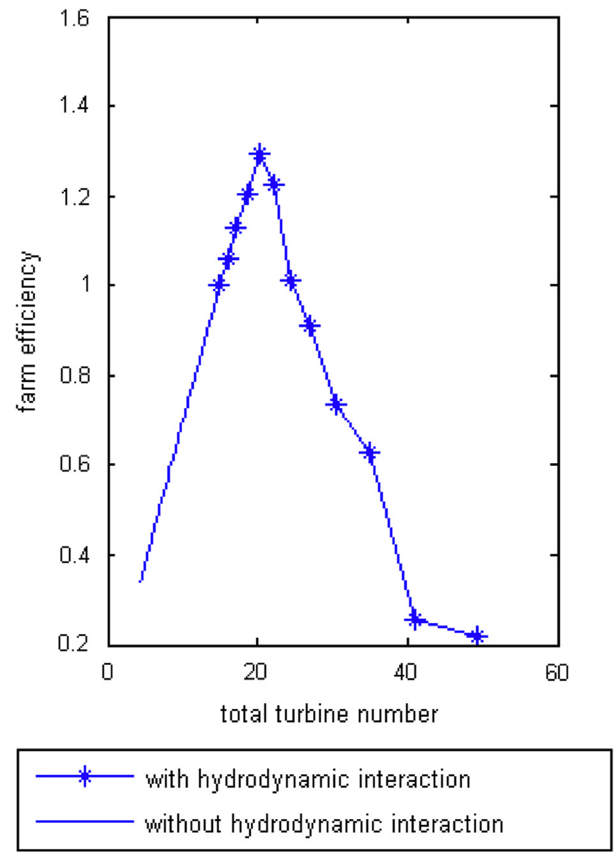

Fig. 8. Farm efficiency of a linear farm.

Eq. (22) is very important to the definition of the farm efficiency. One can solve it using the numerical model developed by Li and Calisal [19,20] or a more precise but more costly method such as Large Eddy Simulation [21]. With the effective distance, one can obtain the total turbine number of a reference farm. We call the total turbine number of the reference farm the reference turbine number, N_{Ref}. It can be obtained as shown in Eq. (26). Thus, the definition of the farm efficiency, Eq. (19), can be re written as Eq. (27).

$$\begin{cases} N_{\mathrm{Ref}} = \max(N) \quad N = 1,2,3\cdots \\ Dr_{i,j} > d_e \quad i,j \in N \end{cases} \tag{26}$$

$$\eta = \frac{P_{\mathrm{farm}}}{N_{\mathrm{Ref}} \times P_S} \tag{27}$$

A farm planner can optimize the turbine distribution by utilizing the turbine wake hydrodynamic interaction and obtaining the maximum power output of the farm site. The maximum farm efficiency can be calculated as the ratio of exact maximum power output to the reference power output, given as Eq. (28).

$$\eta_{\max} = \frac{\max\left(P_{\mathrm{farm}}\right)}{N_{\mathrm{Ref}} P_S} \tag{28}$$

One may note that we have to employ some optimization techniques in seeking the reference turbine number in Eq. (26) and the maximum farm power output in Eq. (28), respectively. As

[5] We assume that all turbines here are identical so that their power outputs are the same. In reality, they can be designed differently due to strong flow fluctuation and other reasons.

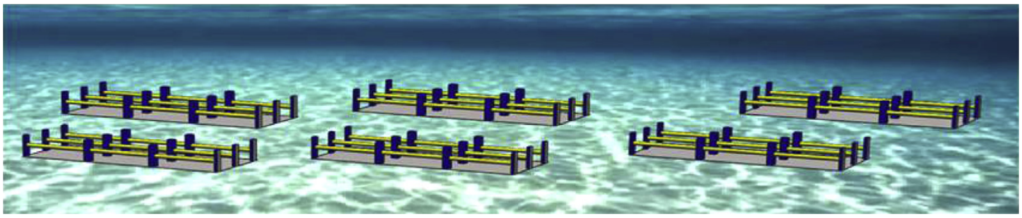

Fig. 9. An illustration of Vortex Induced Vibration Aquatic Clean Energy device (Courtesy of Professor Bernitsas at University of Michigan).

optimization is beyond the scope of this study, we leave it for further studies. Nonetheless, in order to illustrate the definition, we calculate a simple site so that a simple search algorithm can finish the optimization. Particularly, we discuss a farm in a very narrow channel so that the farm demonstrates a line of turbines. We can obtain the reference turbine number as Eq. (29),

$$N_{\mathrm{Ref}} = \frac{L}{d_e} \tag{29}$$

where L denotes the length of the channel. The farm specifications including that the channel length is 1250 m, the incoming flow velocity at the inlet of the channel is 2 m/s. The specifications of the turbine includes that a three-blade vertical axis rotor, with the blade type as NACA0015, the solidity as 0.375, and the Reynolds number as 160,000. For those who are interested in the characteristics of this turbine can refer to Li and Calisal [5]. By using Eqs. (22) and (29), we can find the farm efficiency with respect to the total turbine number and we can find the reference number of turbines in line is 14. (Fig. 8). In general, the procedure for quantifying the farm efficiency presented here is a good normalization method. It shows the relationship between turbine numbers/distribution and the total power output normalized by the reference power output. It shows that, in most cases, when the turbine number is higher than 22 or less than 14, the farm power output is less than the reference power. When the turbine number is less than 14, the power output is proportional to the turbine number because there is no hydrodynamic interaction between turbines. It is noted that the maximum farm efficiency is 1.31 and it can be obtained when the distance between two-neighbor turbines is 13 turbine diameters. It suggests that optimal utilization of the hydrodynamic interactions between turbines can improve the farm power output.

4. Discussion

This paper presents a new definition of power coefficient of tidal current turbine systems to evaluate the reference power of ducted and unducted turbines together based on the same criterion. Specifically, we suggest that the power coefficient of a ducted tidal current turbine shall be quantified by defining a figure of merit for the duct shape. Examples are presented to illustrate the new definition about a ducted turbine with a parabolic wall contour in Fig. 4. The new power coefficient definition can facilitate the standardization of the power efficiencies of various turbine designs and align the discussions between different organizations. This new definition is not a panacea for all devices. An illustration of the difficulties associated with this peculiarity is a vortex induced vibration device such as the Vortex Induced Vibration Aquatic Clean Energy (Fig. 9). More investigations of these new devices should be conducted in the future. Additionally, we stress the importance of adopting the new definition for the ducted turbine. A better definition could be developed in the future.

Furthermore, we also present a definition to quantify the efficiency of a tidal current turbine farm. We find that a farm with hydrodynamic interactions between turbines may produce 30% more power output than that of the farm without hydrodynamic interactions. Poor planning can result in decreased power output. The new definition can help the farm planner to design the farm. Nonetheless, as stated in Eqs. (1) and (2), the key factor for determining if a farm is cost-effective is the cost of energy, and the part related to power is the total annual energy output.

Another interesting point about the farm efficiency is that we did not discuss the impact of change of the current velocity. During a tidal cycle, the current velocity varies with time; thus, the Reynolds number that a farm is operating varies with the velocity. The configuration of the farm with the maximum power output at certain Reynolds number (current velocity) may not obtain its maximum power at a different Reynolds number. Consequently, the industry requires a quantification and measurement of the energy output of a farm. That is, Eq. (19) shall be evaluated with a time integral. Particularly, the reference turbine number is determined by the Reynolds number (current velocity) because the hydrodynamics interaction's impact on the power output is affected by the current velocity and direction. Mathematically, this problem can be formulated as Eq. (30). Consequently, the effective distance in Eq. (22) will be written as a function of time and given as Eq. (31). We consider this topic as a future investigation as well.

$$\begin{cases} \max\left(\int N \times P_S(t)\mathrm{d}t\right) & N = 1,2,3\cdots \\ Dr_{i,j}(t) > d_e(t) & i,j \in N \end{cases} \tag{30}$$

$$d_e = \mathbb{F}(\psi, t, D_r, \mathrm{TSR}, \text{turbine design parameters}, U_\infty, \text{relative rotational direction}) \tag{31}$$

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Appendix

We use the new definition to calculate the power coefficient of the parabolic ducted turbine in Section 2.1. Here we shall show the detailed derivation of the power coefficient. According to the definition of Eq. (8), the frontal area of any given cross-section, the radius of which is l, in the duct can be written as Eq. (32).

$$\mathbb{A}(x) = \pi l^2 \tag{32}$$

Then, by substituting Eq. (13) into Eq. (32), we can get the area of any give cross-section as Eq. (31),

$$\mathbb{A}(x) = \pi\left(ax^2 + r\right)^2 \tag{33}$$

Now, we substitute Eq. (33) into Eq. (11), we can obtain the new reference power as Eq. (35).

$$\begin{aligned} P_{\text{ref}} &= \rho\pi\frac{\left(aL_1^2 + r\right)^6 U_\infty^3}{2(L_1 + L_2)} \int_{-L_1}^{L_2} \frac{1}{\left(ax^2 + r\right)^4} dx \\ &= \rho\pi\frac{\left(aL_1^2 + r\right)^6 U_\infty^3}{2(L_1 + L_2)} (f_1 + f_2)\Big|_{-L_1}^{L_2} \end{aligned} \tag{34}$$

where f_1 and f_2 are given in Eqs. (35) and (36).

$$f_1 = \frac{5 \times \arctan\left(x\sqrt{\frac{a}{r}}\right)}{16r^{3.5}\sqrt{a}} + \frac{5x}{16r^3\left(r + ax^2\right)} \tag{35}$$

$$f_2 = \frac{5x}{24r^2\left(r + ax^2\right)^2} + \frac{x}{6r\left(r + ax^2\right)^3} \tag{36}$$

Finally, by substituting Eq. (14) into Eq. (34), we can have the final result of the new reference power as given in Eq. (37) which is shown as Eq. (15) in Section 2.1.

$$\begin{aligned} P_{\text{ref}} = \rho\pi\frac{\left(aL^2 + r\right)^6 U_\infty^3}{8L} \Bigg(&\frac{5 \times \arctan\left(L\sqrt{\frac{a}{r}}\right)}{8r^{3.5}\sqrt{a}} + \frac{5L}{8r^3\left(r + aL^2\right)} \\ &+ \frac{5L}{12r^2\left(r + aL^2\right)^2} + \frac{L}{3r\left(r + aL^2\right)^3} \Bigg) \end{aligned} \tag{37}$$

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