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## Three-dimensional numerical analysis on blade response of a vertical-axis tidal current turbine under operational conditions

By Ye Li,<sup>1,a)</sup> Naveen Karri,<sup>2</sup> and Qi Wang<sup>3</sup>

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Tidal current as a large-scale renewable source of energy has received significant attention recently. The technology used to harvest energy from tidal current is called a tidal current turbine. Although some of the principles of wind turbine design are applicable to tidal current turbines, to ensure long-term reliability in tidal current turbines, designers must consider elements such as cavitation damage and corrosion. Depending on the orientation of axis, tidal current turbines can be classified as vertical-axis turbines or horizontal-axis turbines. Existing studies on the vertical-axis tidal current turbines focus more on the hydrodynamic aspects of the turbine rather than the structural aspects. This paper summarizes our recent efforts to study the integrated hydrodynamic and structural aspects of vertical-axis tidal current turbines. After reviewing existing methods for modeling tidal current turbines, we developed a hybrid approach that combines a discrete vortex method with a finite element method that can simulate the integrated hydrodynamic and structural response of a vertical-axis turbine. This hybrid method was employed to analyze a typical three-blade vertical-axis turbine. The power coefficient was used to evaluate the hydrodynamic performance, and critical deflection was considered to evaluate the structural reliability. A sensitivity analysis was also conducted with various turbine height-to-radius (H/R) ratios. The results indicated that both the power output and failure probability increase with the turbine height, suggesting a necessity for optimal design. The optimization of a three-blade vertical-axis turbine design using the hybrid method yielded a turbine H/R ratio of about 3.0 for reliable maximum power output. © 2014 AIP Publishing LLC.

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### I. INTRODUCTION

This paper presents a numerical investigation of a vertical-axis current turbine's blade response to hydrodynamic loads caused by unsteady flow in operational conditions.<sup>22</sup> The quantification of turbine characteristics is crucial to turbine manufacturers as the industry is currently assessing the viability of current turbine designs for commercial production. To maximize the performance and reliability of a tidal current turbine, it is necessary to understand the hydrodynamic and structural behavior of the device before it is deployed. Monitoring and maintenance may not be able to focus on the most critical blade and turbine stress points if the analysis of these components is not conducted in advance of deployment. That is, the turbine may fail if a thorough pre-deployment assessment is not conducted.

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Recently, several numerical and experimental investigations were conducted to quantify turbine characteristics such as power output, torque fluctuation, acoustics emission, and induced velocity.<sup>1-3</sup> However, they all focused on hydrodynamics aspect of the turbine. Until now, there has not been a systematic numerical investigation on the structural aspect of the turbine. The structural characteristic is very important for reliable turbine designs. At this stage, the optimal design of tidal current turbines is still under investigation. Because the scale of the device is highly dependent on the structural integrity, the designer must consider both the hydrodynamic and structural characteristics of the turbine for design optimization.

To obtain a comprehensive understanding of turbine blade behavior, we extended our recent hydrodynamic analysis<sup>1</sup> with a structural analysis of a series of three-bladed vertical-axis tidal current turbines (Fig. 1). The analysis of blade response under operational conditions is summarized in this paper. After reviewing existing numerical methods, we developed a hybrid approach that combines a discrete vortex method (DVM) with a finite element method to simulate the turbine behavior. We then used this method to study the blade response to the unsteady hydrodynamic load. We also conducted a sensitivity analysis to study various H/R ratios on a hypothetical turbine made of the high-strength aluminum 5052-H32 material. The results showed that both the power output and failure probability increased with the H/R ratio. We found that under the operational condition, the turbine will fail when the H/R ratio is greater than 3.0. The objective of this study is to provide designers with new perspectives and insights for developing optimal turbine designs.

## II. NUMERICAL METHOD

During the past decade, many numerical methods were developed to simulate tidal current turbines. Examples include Bahaj *et al.*,<sup>2</sup> Coiro *et al.*,<sup>3</sup> Calcagno *et al.*,<sup>4</sup> Camporeale and Magi,<sup>5</sup> Li and Calisal,<sup>6</sup> and Ponta and Jacovkis.<sup>7</sup> However, most of these focused on hydrodynamic performance prediction. A summary of the different methods used and developed in these studies (blade element momentum method, vortex method, boundary element method, and unsteady

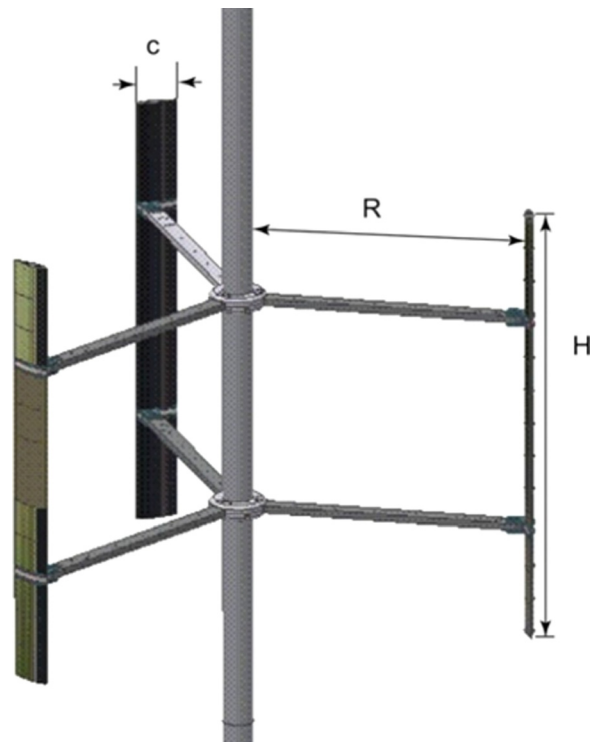


FIG. 1. Sketch of a vertical-axis tidal current turbine (H: turbine height; R: turbine radius; c: chord length).

Reynolds Average Navier-Stokes equations method) can be found in Li *et al.*<sup>8</sup> Among these methods, the discrete vortex method is considered the most suitable method for studying the turbine wake and the boundary element method is considered the most suitable one for studying cavitation. However, these studies did not address the structural aspects of the tidal current turbine.

Recently, a study that combines hydrodynamics and structural dynamics was presented by Young *et al.*<sup>9</sup> This study used a boundary element method and finite element method hybrid approach to simulate horizontal-axis tidal current turbines. Although this approach is well suited for horizontal-axis tidal current turbines, where the wake is steady and the near-field effects are more significant than the far-field effects, it is not suitable for vertical-axis turbines that have highly unsteady wakes. To fully understand the structural and hydrodynamic response of the vertical-axis tidal current turbine with an unsteady wake, we developed a new approach by combining the discrete vortex method with the finite element method.

### A. Vortex-finite element analysis (FEA) methods used in this study

In this study, we used the discrete vortex method developed by the University of British Columbia (DVM-UBC). DVM-UBC was developed based on the traditional discrete vortex method and a potential flow method proposed by Rosenhead.<sup>10</sup> We did not use the traditional DVM because it cannot precisely represent the motion of marine applications caused by viscous effects. By using perturbation theory, Li and Calisal<sup>11</sup> introduced the viscous effect into the traditional DVM and extended it to DVM-UBC by assuming that the viscous effect is limited to the near field where a no-slip boundary condition exists.<sup>6</sup> DVM-UBC replaces the potential vortices used in the traditional DVM with Lamb vortices and introduces the vortex decay scheme into the formula for the life cycle of vortices that approximates the physics of the unsteady flow around the turbine. One set of bounding vortices is employed to represent the blades, and the other set of free vortices, together with uniform flow, is used to represent the unsteady wake (Eqs. (1) and (2)). The arm and shaft effects on the power coefficient of the turbine can be given as a cubic function, and an example of the arm and shaft effect on the power coefficient is given in Eqs. (3) and (4).<sup>12</sup> DVM-UBC is a time-dependent method. In each time step, it utilizes the relationship between the strength of the free vortex and the induced velocity to approximate the flow and predict the lift. It then uses this relationship together with the viscous effects to predict the drag. The lift and drag are then used to calculate instantaneous power. For the proposed approach, these lift and drag forces are fed to the structural analysis model for instantaneous blade response.

In general, the DVM-UBC can predict turbine hydrodynamic behavior with good accuracy and low computational cost. It has been well validated with experimental testing and verified with a high-fidelity computational fluid dynamics (CFD) tool.<sup>6,12</sup> It has been also used to study various aspects of tidal current turbines, including interactions between turbine blades and between multiple turbines.<sup>13–15</sup> Therefore, we may restate some descriptions that were used in those studies in this section

$$\Gamma_{S,i-1,j} = \Gamma_{B,i-1,j} - \Gamma_{B,i,j}, \quad (1)$$

$$\Gamma_{T,i-1,j} = \Gamma_{B,i,j} - \Gamma_{B,i,j-1}, \quad (2)$$

where  $\Gamma_B$ ,  $\Gamma_S$ , and  $\Gamma_T$  represent the strengths of bounded vortex, the spanwise vortex filament, and the trailing edge vortex shedding filament, respectively, and  $i$  is the index of the time step and  $j$  is the index of the blade element

$$C_{P,AS} = \kappa_1 TSR^3 + \kappa_2 TSR^2 + \kappa_3 TSR + \kappa_4, \quad (3)$$

$$C_P = C_{P,blade} + C_{P,AS}, \quad (4)$$

where  $C_P$ ,  $C_{P,AS}$ , and  $C_{P,blade}$  denote the turbine power coefficient, the arm and shaft effects on turbine power coefficient, and the turbine power coefficient with blade only, respectively.  $TSR$

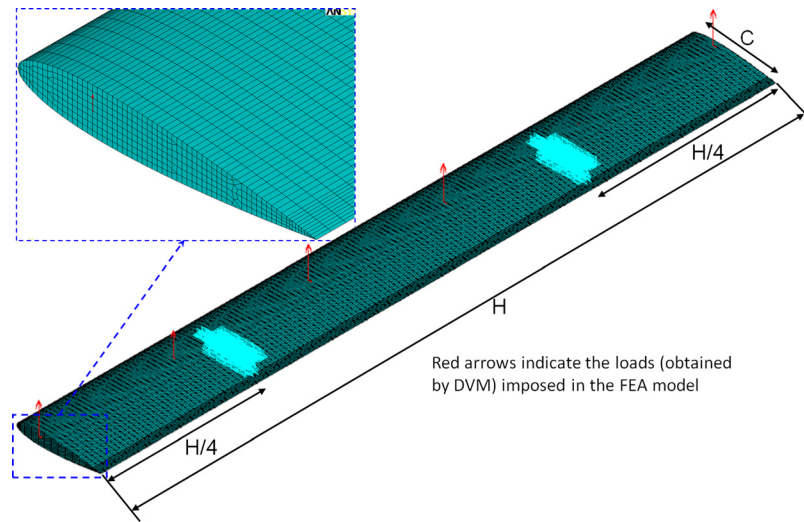


FIG. 2. Finite element model of the blade with  $H/R = 1$ .

denotes the tip speed ratio that is the ratio of the blade tip speed to the incoming flow speed. The parameters  $\kappa_i$  ( $i = 1, 2, 3$ , and  $4$ ) are determined by the profile of the arm's cross section, the material of the arm, and the Reynolds number.<sup>12</sup>

The FEA in this study was conducted using the commercial finite element code ANSYS® version 12.0. Fig. 2 shows the finite element model of a NACA0015 profiled turbine blade with  $H/R = 1$ . The blade geometry is modeled with eight node hexahedral elements with full integration.<sup>23</sup> The blade is supported (fixed) at two locations that are  $1/4$  of the blade's length from each end to simulate the fixed boundary conditions shown in Fig. 1.

### III. NUMERICAL PREDICTION

In this section, we present the blade response obtained with the DVM-FEA hybrid approach proposed in Sec. II. The basic specifications of the turbine used in this study include a three-bladed vertical-axis turbine with an NACA0015 blade profile. In this study, the radius ( $R$ ), the turbine height ( $H$ ), and the chord length ( $L$ ) of the blades are 750 mm, 750 mm, and 93.75 mm, respectively. As the purpose in this study was to analyze blade reliability, we used a very strong material for the blade; a high-strength aluminum 5052-H32 with a Young's Modulus 70.3 GPa, a Poisson's ratio of 0.33, a density at 2800 kg/m<sup>3</sup>, and a yield strength of 193 MPa.

Although a turbine can operate with various tip speed ratios, to keep the analysis focused on the critical scenarios, we present the blade response when the turbine achieves its maximum power output, and the tip speed ratio is 3.75. Because the power output of vertical-axis tidal current turbines was extensively studied and presented in previous works, we present only the structural results in this study. However, the power output together with the turbine blade durability considerations are discussed in Sec. V.

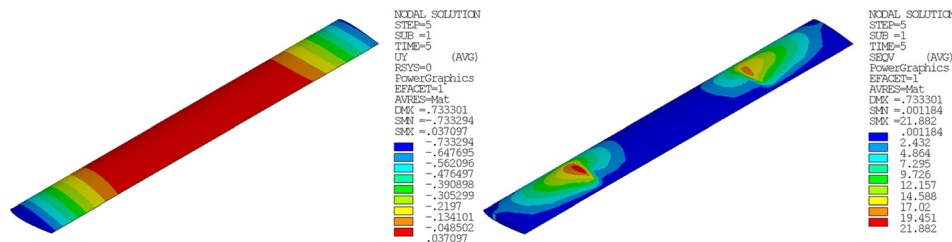


FIG. 3. Normal stage (a) deflection (mm) and (b) von Mises stress (MPa).

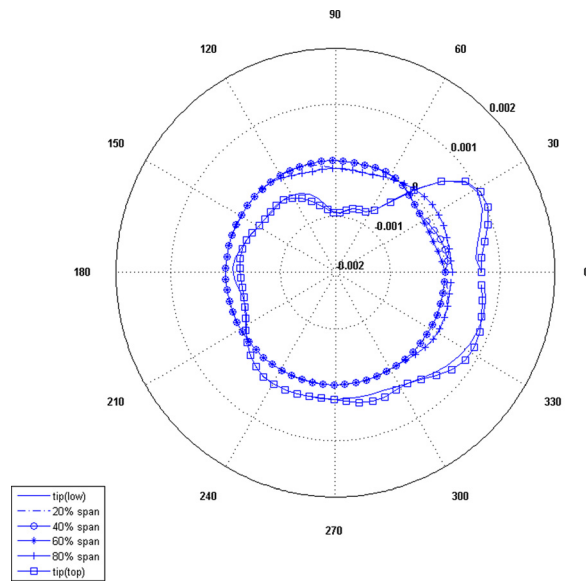


FIG. 4. Blade deflection along the span.

Fig. 3(a) shows the normal deflection (in mm) of the blade under the combination of maximum normal and tangential forces that are experienced at any time during a rotation. In these figures, the normal deflection (UY) is in the direction normal to the chord length. Fig. 3(b) shows the von Mises stresses (SEQV) corresponding to the deflections in Fig. 3(a). The maximum stresses (~21 MPa) were noticed on the surface (skin) at locations where the blade is fixed, and these stresses are below the yield strength (193 MPa) of the material used in the study.

Figure 4 shows the dimensionless deflections of the blade in one complete rotation. Here, the deflections are normalized with respect to blade length (H). It is noted that the blade undergoes a maximum deflection of about 0.001 when the blade is at the azimuth angle of 30° or 330°. An interesting feature here is that the blade top tip deflects stronger than the lower tip. This is the asymmetric flow structure of the turbine wake, and one can refer to Ref. 12 for more discussion.

To determine whether the blade is overloaded and allow us to discuss the result nondimensionalized by turbine height, we define a critical deflection that produces a maximum stress equivalent to the yield stress of the blade material in tension (or compression).<sup>24</sup> This methodology also provides a feel for the magnitude of deflections required for blade failure under varying H/R ratios. The blade was loaded with displacement controlled loading and the maximum von Mises stress was monitored until the critical point when the stress in the blade reached the yield strength of the material. A detailed contour of the blade deflection at the critical stage and its corresponding von Mises stress is given in Fig. 5. The maximum von Mises stress that is nearly equal to the yield strength of the material occurs at the fixed supports.

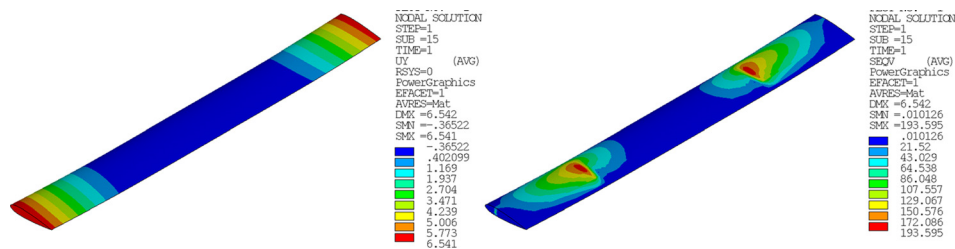


FIG. 5. Critical stage (a) deflection (mm) and (b) von Mises stress (MPa).



More importantly, it is noted that the critical deflection is 0.0087, which is much higher than the maximum deflection of the blade, 0.001. Because the maximum deflection of the blade at any instance in one complete rotation is less than the critical deflection of the blade for the  $H/R = 1$  case, the blade is considered to be durable under operational conditions with the material considered in this study.

#### IV. RELATIONSHIP BETWEEN BLADE DEFLECTION AND HEIGHT-TO-RADIUS RATIO

We studied the blade deflection when the turbine height was equal to the turbine radius, i.e.,  $H/R$  ratio was equal to one in the last section, and this showed that the turbine is safe

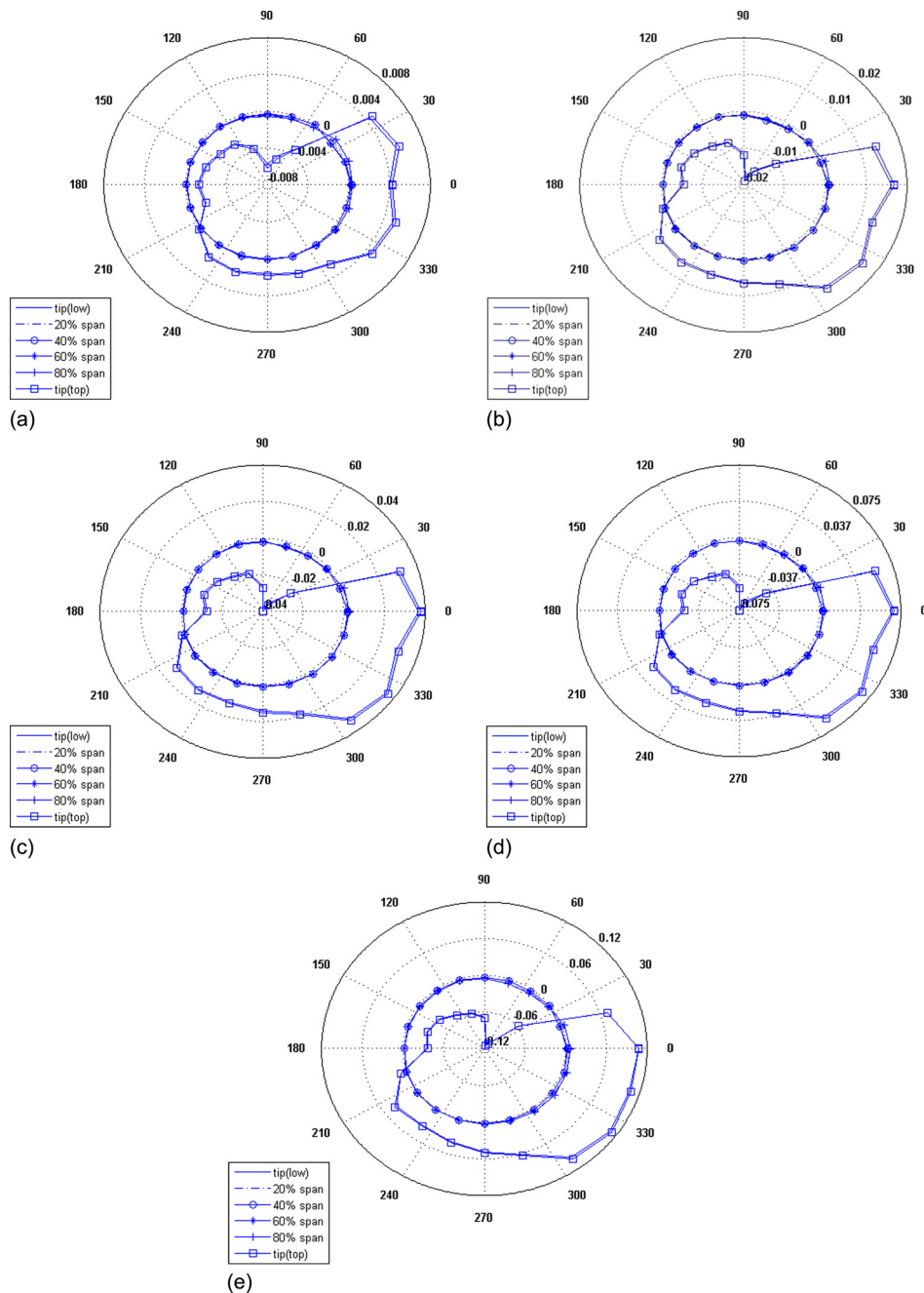


FIG. 6. Relationship between height-to-radius ratio and blade deflection: (a)  $H/R = 2.0$ ; (b)  $H/R = 3.0$ ; (c)  $H/R = 4.0$ ; (d)  $H/R = 5.0$ ; and (e)  $H/R = 6.0$ .

under operational conditions. From the structural point of view, the higher the H/R ratios, the higher the forces on the blade. Hence, the maximum value of this H/R ratio for any specific blade design is limited by the mechanical strength of the blade, which in turn depends on the geometry and material properties of the blade. To understand the influence of the H/R ratio on turbine performance, a systematic analysis of the blade deflection in several scenarios with varying H/R ratios was conducted following the same procedure as in the H/R = 1 case described in Sec. III. Specifically, while keeping other parameters the same, we studied the scenarios when H/R is equal to 2.0, 3.0, 4.0, 5.0, and 6.0. In all the cases, the number of blades is 3, the blade profile is NACA0015, the radius is 750 mm, and the chord length is 93.75 mm. Because the radius was kept constant, the corresponding heights of turbine for above H/R ratios are 1500 mm, 2250 mm, 3000 mm, 3750 mm, and 4500 mm, respectively.

Figures 6(a) through 6(e) show the resulting blade deflections along the span in one complete cycle. In these results, the H/R ratio was varied from 2.0 (Fig. 6(a)) through 6.0 (Fig. 6(e)). Note that these results were obtained when the maximum power of the turbine was obtained, and the specific corresponding power outputs are shown in Sec. V. As seen in the earlier case with H/R = 1 (see Fig. 4), the blade deflection at its tip is much greater than at other places along the span. It is noted that the azimuth angle at which maximum deflection is seen decreases when the H/R ratio increases. When the H/R ratio is greater than 3.0, the maximum deflection was observed when the azimuth angle is close to 0°. This is because, the greater the H/R ratio, the less the three-dimensional effect in the hydrodynamic load.<sup>12</sup>

The maximum deflection of the blade for each of the H/R ratios along with the corresponding critical deflection is summarized in Table I. When the height-to-radius ratio is equal to 2.0, the maximum deflection is 0.0056, and the maximum stress in the blade is 88 MPa. These values are much less than the critical deflection and the critical stress (193 MPa). This case is considered to be durable for turbine operation without the risk of blade failure. When the H/R ratio is equal to 3.0, the maximum deflection of the blade is close to the critical deflection, and the maximum stress is close to the critical stress. Hence, any sudden gusts may lead to turbine failure. Compared to normal operational conditions, a gust could cause a 150% overload on the device. When the height-to-radius ratio is greater than 3.0, the maximum deflection is always greater than the critical deflection that leads to turbine failure.

## V. DISCUSSION

This paper presents a hybrid approach that combines a discrete vortex method and a finite element method to study the blade response of a vertical-axis tidal current turbine under operational conditions. We studied the blade response of vertical-axis tidal current turbines with a focus on blade failure. Turbines with various H/R ratios were investigated. Particularly, in order to study the maximum blade deflection, we selected a high-strength material, aluminum 5052-H32 for the blade. In this section, we present some insights.

Although we only discussed the structural response of the turbine in Secs. III and IV, an integration analysis combining both hydrodynamic power output and structural response is very important to the cost-effectiveness of the turbine. Fig. 7 shows the relationship between the blade deflection and the H/R ratio as well as the relationship between the H/R ratio and power

TABLE I. Summary of the maximum deflection and critical deflection.

H (mm)	H/R	Normalized maximum deflection	Normalized critical deflection
750	1	0.001	0.008
1500	2	0.006	0.013
2250	3	0.017	0.018
3000	4	0.038	0.023
3750	5	0.069	0.028
4500	6	0.108	0.034



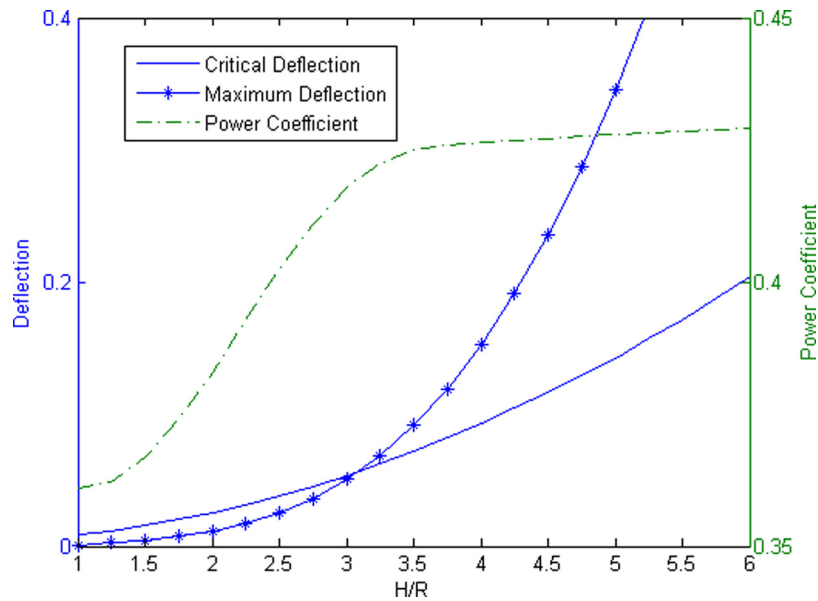


FIG. 7. Relationship between H/R ratio and turbine characteristics.

coefficient. In this figure, unlike the discussions in Secs. III and IV, as turbine height is a variable, we normalized the blade deflection with respect to the turbine radius. Such a plot is important from the decision-making perspective to understand the scale and size of the turbine. It is clear from this plot that the power coefficient increases with H/R ratio, and, hence, a higher H/R ratio is desired. However, the plot also indicates that the maximum deflection crosses the critical deflection for the H/R ratios beyond 3.0. Hence, there is a tradeoff between high performance and reliability. Based on the preliminary analysis, presented in this article, we recommend that the design of any horizontal-axis turbine should consider an optimal H/R ratio while maintaining a high power coefficient along with structural stability. Such a decision-making process should also include a cost analysis of the turbine material because one has to maintain a balance between low capital cost and low operational and maintenance cost. For the blade configuration and material considered in our study, the turbine H/R ratio cannot be greater than 3.0.

In addition to deflection, resonance is an important aspect related to blade reliability. The first five fundamental frequencies associated with the various turbine configurations (H/R ratios) considered in this study are evaluated and listed in Table II. It is evident from these results that the fundamental frequencies are significantly affected when the H/R ratios change. The frequencies decrease with increased H/R ratios, as one would expect. In this table, the frequencies are rounded off to the nearest integer. As H/R increases, the first and second modes of the frequencies approach a same value with differences appearing in the second or third decimal. In the first mode, both ends of the blade vibrate (in phase) up or down, whereas in second mode, the ends vibrate in opposite directions (out of phase). When the H/R ratio is 6.0, the fundamental

TABLE II. The first 5 natural frequencies of the blade for various H/R ratios.<sup>25</sup>

H/R→	1	2	3	4	5	6
Freq 1 (Hz)	200	62	30	17	11	8
Freq 2 (Hz)	223	63	30	17	11	8
Freq 3 (Hz)	402	106	49	28	18	13
Freq 4 (Hz)	1038	286	136	78	50	35
Freq 5 (Hz)	1290	389	186	108	70	48

frequency of the blade is 8 Hz, which indicates a very low resonant frequency. The turbine blade might be at risk of failure as the frequency is close to some ambient fluctuation such as turbulence. Blade failure could occur either from resonant frequencies causing severe permanent deflection or from insufficient material strength under fatigue leading to blade fracture under cyclic hydrodynamic loads. Therefore, it is very important for a designer to evaluate the ambient incident loads. We also conducted the eigenvalue analysis by using a beam solver, geometrically exact beam theory (GEBT),<sup>16,17</sup> to further reduce the computational cost. The first natural frequency with  $H/R = 1$  obtained by GEBT is 191, where the percentage difference in comparison with the three-dimensional (3D) result listed in Table II is 4.5%. The computational time for the ANSYS 3D analysis is 48 s and only 0.256 s in GEBT. Both cases were run on a desktop personal computer with AMD FX(tm)-8150 3.60 GHz processor and 16.0 GB RAM.

Finally, it must be noted that the failure criterion used in these analyses might not be applicable to all materials. In this analysis, the von Mises criterion was used, as the blade material considered is aluminum, which is ductile. If materials such as titanium and other brittle materials or advanced composites are used in blade manufacturing, it is necessary to consider an applicable failure criterion that is more robust and specific to those specific blade designs.

## VI. FUTURE WORK

The main purpose of this research is to evaluate the blade response of a vertical-axis tidal current turbine by developing a new hybrid approach, i.e., DVM-FEA, to simulate the dynamics of the vertical-axis tidal current turbine. Although the DVM-FEA provides a more comprehensive analysis, as this is the first attempt using simplified assumptions, many challenges still exist. Based on these results, we recommend the following for future studies:

- Many turbines are expected to operate close to the free surface, but the free surface effect is not included in this study. Additional description of the free surface should be introduced in the current formulation of the discrete vortex method. We believe that combining the boundary element method and the discrete vortex method may provide an effective solution.<sup>18</sup>
- The material used in this analysis is high-fatigue-strength aluminum. There are other alternative materials with high-corrosion resistance, as well as fatigue and yield strengths suitable for marine applications. We recommended running similar analyses with these high-strength materials.
- Compared to the 3D solid model, using a beam model for the analysis of tidal turbines can reduce the computational cost by two to three orders of magnitude as discussed in Sec. V. We recommend studying the dynamic behavior of the tidal turbine system based on a new methodology for beam analysis that considers the geometrical nonlinearity and complicated cross-sections that include composite materials.<sup>19</sup>
- In this study, we only analyzed the loads on the blade. The loads on the arms could be critical as well if there is significant shear. However, before an arm analysis is conducted, more real world measurement and quantification of incoming flow shear profile or a turbulence spectrum analysis should be conducted.
- This study focused on the turbine performance under operational conditions. If real world turbulence data are available, we recommend a study of turbine performance under extreme conditions. The survivable conditions load is usually the load used for optimizing a turbine design if the survivable conditions are much harsher than the operational conditions.<sup>20,21</sup>

## ACKNOWLEDGMENTS

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- <sup>22</sup>Note that when turbine is mentioned in this paper, we mean vertical-axis tidal current turbine unless otherwise specified.
- <sup>23</sup>Note the node here is totally different from the blade element in the fluid mechanics treatment, which is based on the continuity.
- <sup>24</sup>It is worth noting that there is an alternative way to produce the critical scenario analysis with more details. One can calculate the stress based on the real load and then analyze the stress with the strength. However, this approach does not allow us to nondimensionlize the results with the turbine height.
- <sup>25</sup>Note for the first two frequencies, the corresponding values when H/R equal to 3-6 are similar due to the round effect. The frequency 1 is 29.93, 17.24, 11.14, and 8.15 when H/R equal to 3-6. The frequency 2 is 30.01, 17.25, 11.146, and 8.152 when H/R equal to 3-6. To keep the integrity of the article, we do not extend in the text.